Self-organization in large scale ad hoc networks

Nathalie Mitton, Anthony Busson, Eric Fleury

Self-organization in large scale ad hoc networks

Nathalie Mitton
CITI/ARES - INRIA
Villeurbanne F-69621
nathalie.mitton@insa-lyon.fr

Anthony Busson
IEF - CNRS UMR 8622
Orsay F-91405
anthony.busson@ief.u-psud.fr

Eric Fleury
CITI/ARES - INRIA
Villeurbanne F-9621
eric.fleury@inria.fr

***Abstract*— Flat ad hoc architectures are not scalable. In order to overcome this major drawback, hierarchical routing is introduced since it is found to be more effective. The main challenge in hierarchical routing is to group nodes into clusters. Each cluster is represented by one cluster head. Conventional methods use either the connectivity (degree) or the node Id to perform the cluster head election. Such parameters are not really robust in terms of side effects. In this paper we introduce a novel measure that both forms clusters and performs the cluster head election. Analytical models and simulation results show that this new measure for cluster head election induces less cluster head changes as compared to classical methods.**
Keywords: ad hoc, sensors, wireless, self-organization, stochastic geometry, Palm distribution, scalability

I. Introduction

Wireless ad-hoc networks consist of a set of mobile wireless nodes without the support of a pre-existing fixed infrastructure. Ad hoc networks have applications in battlefields coordination or on-site disaster relief management. Each host/node acts as a router and is able to arbitrary move. This feature is a challenging issue for protocol design since the protocol must adapt to frequent changes of network topologies. More recently, researchers apply ad hoc paradigms in sensor networks which induce to be able to set up a very large number of nodes.

In order to be able to use ad hoc networks on very large scale, flat routing protocols (reactive or proactive) are not really suitable. Indeed, such routing protocols become ineffective for large scale wireless networks, because of bandwidth (flooding of control messages) and processing overhead (routing table computation). One well known solution to this scalability problem is to introduce a hierarchical routing by grouping geographically close nodes into clusters and by using an "hybrid" routing scheme: classically proactive approach inside each cluster and reactive approach between clusters ([12], [14]). Such an organization also presents numerous advantages as to synchronize stations in a group or to attribute new service zones more easily.

In this paper we propose a new metric suitable for organizing an ad hoc network into clusters and we propose a new distributed cluster head election heuristic. Our new metric does not rely on "static" parameters and thus our novel heuristic extends the notion of cluster formation. The proposed heuristic allows load balancing to insure a fair distribution of load among cluster heads. Moreover, we implement a mechanism for the cluster head election that tries to favor their re-election in future rounds, thereby reducing transition overheads when old cluster heads give way to new ones. We expect a network organization to be robust towards node mobility. If we want to keep overhead as low as possible, our organization must change as less as possible when nodes move and topology evolves. Moreover, we would like to be able to apply some localization process and inter-groups routing above our organization.

The remainder of this paper is organized as follows. Section II defines the system model and introduces some notations. Section III reviews several techniques proposed for cluster head selection. Sections IV and V will present our main contribution and will detailed the distributed selection algorithm and give a formal analysis. Simulation experiments presented in Section VI demonstrate that the proposed heuristic is better than earlier heuristics. Finally, we conclude in Section VII by discussing possible future areas of investigation.

II. System model

In an ad hoc network all nodes are alike and may be mobile. There is no base station to coordinate the activities of subsets of nodes. Therefore, all the nodes have to collectively make decisions and the use of distributed algorithms appears to be mandatory. Moreover, all communication are performed over wireless links. As usual, we model an ad hoc network by a graph $G = (V, E)$ where V is the set of mobile nodes ($|V| = n$) and $e = (u, v) \in E$ represents a wireless link between a pair

of nodes u and v only if they are within communication range of each other.

For the sake of simplicity, let first introduce some notations. Let call $d(u, v)$ the euclidean distance between nodes u and v. We note $\mathcal{C}(u)$ the cluster owning the node u and $\mathcal{H}(u)$ the cluster head of this cluster. From graph theory textbook [8] we will also note $\Gamma_k(u)$ the *k-neighborhood* of a node u, *i.e.*, $\Gamma_k(u) = \{v \in V | 0 < d(u, v) \leq k\}$ and will note $\delta_k(u) = |\Gamma_k(u)|$.

We will note $e(u/\mathcal{C}) = \max_{v \in \mathcal{C}(u)}(d(u, v))$ the *eccentricity* of a node u inside its cluster. Thus the *diameter* of a cluster will be $D(\mathcal{C}(u)) = \max_{v \in \mathcal{C}(u)}(e(v/\mathcal{C}))$.

III. Related work

Researchers have proposed several techniques for cluster formation and cluster head selection. All solutions aim to identify a subset of nodes within the network and bind it a leader. Each clusterhead is responsible for managing communication between nodes into clusters as well as routing information to other clusterheads in other clusters. Typically, backbones are constructed to connect neighborhoods in the network.

Past solutions try to gather nodes into homogeneous clusters by using either an identity criteria (*e.g.*, the lowest Id [9], [3]) or a fixed connectivity criteria (maximum degree [4], max-min D-cluster [1], 1-hop clusters [2], [5], [11], k-hop clusters [6]).

Such solutions based on a fixed cluster diameter [7], [4], [1], fixed cluster radius [13] or a constant number of nodes [15] are not adapted to large ad hoc networks since they may generate a large number of clusterheads. Therefore, it is suitable to control the clusterhead density in the network. Note that some previous clustering solutions also rely on synchronous clocks for exchange of data between nodes. It is the case for example in the Linked Cluster Algorithm [2], but such an heuristic is developed for relatively small number of nodes (less than 100). Solutions based on the degree or the lowest Ids can result in a high turnover of clusterheads when topology changes [10], [7]. Solutions were also envisaged to base the election on a pure mobility criteria [4] but if mobility should be taken into account, electing only non mobile nodes may result in isolated cluster heads, which may be useless.

In all previous works, the design of clusters selection appears to be similar with few variants. Each node locally computes its own value of a given criteria (degree, mobility...) and locally broadcasts this value in order to compete with its neighbors. All nodes are thus able to decide by their own if they win the tournament and can be declared cluster head. In case of multiple winners, a second criteria (*e.g.*, Id) is used.

IV. Main objectives

The main goal is to design a heuristic that would select some nodes as clusterheads and computes clusters in a large ad hoc network. As we mentioned in the previous section, the definition of a cluster should not be defined a priori by some fixed criteria but must reflect the *density* of the network. In order to be scalable, the heuristic should be completely distributed and asynchronously avoid any clock synchronization. The number of messages exchanges should be minimized. In fact, we use only local broadcast messages like HELLO PACKET in order to discover the 2-neighborhood of a node. Finally, in order to ensure stability, it would be better not to re-elect cluster head whenever it is possible and nodes that are ”too” mobile to initiate any communication will not participate in the ballot phase.

The criteria metric should gather and aggregate nodes into clusters not on an absolute criteria (like degree or diameter) and thus should be adaptive in order to reflect the features of the network. To elect a cluster head, we need to promote node stability by limiting traffic overhead when building and maintaining the network organization. Secondly, the criteria should be robust, *i.e.* not be disturbed by a slightly topology change. Finally, the criteria should be computed locally by using only local traffic (intra-cluster routing) since it is cheaper that inter-cluster traffic. The main goal is to recompute the clusters topology as less as possible in spite of great nodes mobility.

Based on these general requirements we propose a novel heuristic based on a metric criteria which gathers the density of the neighborhood of a node. This density criteria reveals to be stable when the topology slightly evolves. As the network topology slightly changes the node’s degree is much more likely to change than its density, which smoothes the relative topology changes down inside its own neighborhood.

V. Our contributions

A. The density metric criteria

In this section, we introduce our criteria called *density*. The notion of density should characterize the ”relative” importance of a node in the ad hoc network and in its k-neighborhood. As mentioned earlier, the node degree is not adequate. The density notion should absorb small topology changes. The underlying idea is that if some

nodes move in $\Gamma_1(u)$ (*i.e.*, a small evolution in the topology), changes will affect the microscopic view of node u (its degree $\delta_1(u)$ will change) but its macroscopic view will in fact not change since globally the network does not drastically change and its $\Gamma_1(u)$ globally remains the same. The density is directly related to both the number of nodes and links in a k-neighborhood. Indeed, the density will smooth local changes down in $\Gamma_k(u)$ by considering the ratio between the number of links and the number of nodes in $\Gamma_k(u)$.

***Definition** 1 (density):* The k-density of a node $u \in V$ is

$$\rho_k(u) = \frac{|e = (v,w) \in E \; v \in \{u, \Gamma_k(u)\} \quad w \in \Gamma_k(u)|}{\delta_k(u)} \tag{1}$$

The 1-density (also noted $\rho(u)$) is thus the ratio between the number of edges between u and its 1-neighbors (by definition the degree of u), the number of edges between u's 1-neighbors and the number of nodes inside u's 1-neighborhood .

In the following, we will see that the most robust metric among these different ones is in fact the 1-density, which is also the cheapest in terms of messages exchanges. Indeed, note that to compute $\rho_k(u)$, the node u must know $\Gamma_{k+1}(u)$ since it must be able to compute the number of edges that exist between all its k-neighbors.

B. Cluster head selection and cluster formation

1) Basic idea: Each node computes its k-density value and locally broadcasts it to all its k-neighbors. Each node is thus able to decide by itself whether it wins in its 1-neighborhood (as usual, the smallest Id will be used to decide between joint winners). Once a cluster head is elected, the cluster head Id and its density is locally broadcasted by all nodes that have joined this cluster. The cluster can then extend itself until it reaches a cluster frontier of another cluster head. The only constraint that we introduce here to define a cluster is that two neighbors can not be both cluster head. This ensures that a clusterhead is not too off-center in its own cluster, that a cluster has at least a diameter of two and that two cluster heads are distant of at least three hops.

2) Heuristic: The heuristic process is quite simple. On a regular basis (frequency of HELLO packets for instance), each node computes its k-density based on its view of its $k+1$-neighborhood. To simplify the notation we describe the 1-density heuristic with algorithm 1. The k-density is similar since the only modification to be made is to gather the $(k+1)$-neighborhood which is given by sending HELLO within k-hops.

Algorithm 1 Cluster head selection

For all node $u \in V$

▷ *Checking the neighborhood*
Gather $\Gamma_2(u)$
if mobility$(u, \Gamma_1(u)) >$ threshold **then**
  ▷ *Checking the 1-neighborhood consistency. If this one changes too much, node u will not participate to the ballot phase since it is "relatively" too mobile.*
  break
end
Compute $\rho(u)$
Locally broadcast $\rho(u)$
▷ *This local broadcast can be done by piggybacking $\rho(u)$ in* HELLO *packets.*
▷ *Node u is aware of all of its 1-neighbors' density value and knows whether they are eligible.*
if $(\rho(u) = \max_{v \in \Gamma_1(u)}(\rho(v)))$ **then**
  $\mathcal{H}(u) = u$
  ▷ *u is promoted cluster head.*
  ▷ *Note that if several nodes are joint winners, the winner will be the previous cluster head whether it exists, otherwise, the less mobile node, otherwise, the smallest Id.*
  $\forall v \in \Gamma_1(u), v \in \mathcal{C}(u)$
  ▷ *All neighbors of u will join the cluster created by u as well as all nodes which had joined u's neighbors.*
else
  ▷ $\exists w \in \Gamma_1(u) | \rho(w) = \max_{v \in \Gamma_1(u)}(\rho(v))$
  $\mathcal{H}(u) = \mathcal{H}(w)$
  ▷ *Either $\mathcal{H}(w) = w$ and u is directly linked to its cluster head, either w has joined another node x and $\mathcal{H}(u) = \mathcal{H}(w) = \mathcal{H}(x)$.*
  ▷ *If there exist k ($k > 1$) nodes w_i such that $\rho(w_i) = \max_{v \in \Gamma_1(u)}(\rho(v))$ and such that $w_i \notin \Gamma_1(w_j)(i \neq j)$ then u will join the node w_i which Id is the lowest and all $\mathcal{C}(w_i)$ (for i=1 to k) will merge consequently.*
end
Locally broadcast $\mathcal{C}(u)$
▷ *Each node will know whether its 1-neighbors belong to the same cluster as it and whether two of its 1-neighbors belong to the same cluster. This will be useful for the routing process.*

C. Maintenance

Given that every node is mobile and subject to move at any time, our cluster organization must adapt to topology changes. For this, our nodes have to periodically check their environment and so check their mobility. If they become too mobile, they will not join any cluster, if at the opposite, they were too mobile and now are able to communicate, they will join the cluster of their neighbor which has the highest density. Each node periodically checks its density and its neighbors' one. They continue

on joining their neighbor which has the highest density. If this last changes, the reconstruction will be automatic without generating much additional traffic overhead.

D. Analysis of the average density

In this section we compute several important characteristic factors of our cluster heuristic. We first compute the mean density of nodes and then we compute an upper bound on the expected number of clusterheads.

In this subsection we analyze the average k-density $\tilde{\rho}_k(u)$ of a node u. We consider a multiple-hop ad hoc network where nodes are distributed according to a Poisson point process of constant spatial intensity λ. Each node has a transmission range equal to R depending on its transmitting power $\mathcal{P}_u$.

In this section, we compute the mean density under Palm probability. We thus compute the density of a node located at the origin point (under Palm probability, there exists almost certainly a point in 0). Since a Poisson process is a stationary process, this node density is valid for every point. Let $\rho(0)$ be the density value of node 0. Φ is used to design the point process. $\mathbb{E}^o$ and $\mathbb{P}^o$ design respectively the expectation and the probability under Palm distribution.

We compute:

$$\tilde{\rho}(u) = \mathbb{E}^o\left[\rho(0)\right]$$

***Lemma** 1:* The mean 1-density of any node u is $\tilde{\rho}(u) = \mathbb{E}^o\left[\rho(0)\right]$ where:

$$\begin{aligned}\mathbb{E}^o\left[\rho(0)\right] = 1 + \frac{1}{2}\left(\pi - \frac{3\sqrt{3}}{4}\right) \\ \times \left(\lambda R^2 - \frac{1 - exp\{-\lambda\pi R^2\}}{\pi}\right)\end{aligned} \tag{2}$$

We detail here the proof for the 1-density.

Proof: Let $B_u^{'}$ be the ball centered in $u \in I\!R^2$, with radius R minus the point u, that is, $B_u^{'} = B(u,R)\backslash u$ where $B(u,R)$ is the ball centered in u with radius R . Let's note $(Y_i)_{i=1,..,\Phi(B'_0)}$ Φ's nodes being in B'_0. From the density definition, we have:

$$\mathbb{E}^o\left[\rho(0)\right] = 1 + \frac{1}{2}\mathbb{E}^o\left[\sum_{i=1}^{\Phi(B'_0)} \frac{\Phi(B'_0 \cap B'_{Y_i})}{\Phi(B'_0)}\right]$$

Moreover, we suppose that $\rho(0) = 1$ if $\Phi(B'_0) = 0$. We consider the expected value conditioned on the number of nodes in B'_0. Thus we have:

$$\begin{aligned}\mathbb{E}^o\left[\rho(0)\right] &= 1 + \frac{1}{2}\sum_{k=1}^{+\infty}\mathbb{E}^o\left[\sum_{i=1}^{\Phi(B'_0)} \frac{\Phi(B'_0 \cap B'_{Y_i})}{\Phi(B'_0)}\Big|\Phi(B'_0) = k\right] \\ &\quad \times \mathbb{P}^o\left(\Phi(B'_0) = k\right) \\ &= 1 + \frac{1}{2}\sum_{k=1}^{+\infty}\sum_{i=1}^{k}\frac{1}{k}\mathbb{E}^o\left[\Phi(B'_0 \cap B'_{Y_i})\Big|\Phi(B'_0) = k\right] \\ &\quad \times \mathbb{P}^o\left(\Phi(B'_0) = k\right)\end{aligned} \tag{3}$$

Moreover, we know that $\Phi(B'_0) = k$, and that nodes $(Y_i)_{i=1,..,k}$ are independent from one to each other and uniformly distributed in B'_0. Thus, $\mathbb{E}^o\left[\Phi(B'_0 \cap B'_{Y_i})\Big|\Phi(B'_0) = k\right]$ is the same for all $i, i = 1,..,k$. Knowing that $\nu(B'_0 \cap B'_{Y_i})$ and $\Phi(B'_0) = k$ (ν is the Lebesgue measure in $I\!R^2$), the amount of nodes in $B'_0 \cap B'_{Y_i}$ follows a binomial law with parameter $\left(k-1, \frac{\nu(B'_0 \cap B'_{Y_i})}{\nu(B'_0)}\right)$ and the mean number of points is $(k-1)\frac{\nu(B'_0 \cap B'_{Y_i})}{\nu(B'_0)}$.

Thus we have, for all $i = 1,..,k$:

$$\begin{aligned}&\mathbb{E}^o\left[\Phi(B'_0 \cap B'_{Y_i})\Big|\Phi(B'_0) = k\right] \\ &= \frac{(k-1)}{\pi R^2}\mathbb{E}^o\left[\nu(B'_0 \cap B'_{Y_i})\Big|\Phi(B'_0) = k\right] \\ &= \frac{(k-1)}{\pi R^2}\mathbb{E}^o\left[\nu(B'_0 \cap B'_{Y_i})\right]\end{aligned}$$

This last equality comes from the fact that the area $\nu(B'_0 \cap B'_{Y_i})$ does not depend on the number of nodes in B'_0, since all Y_i are independent.

Knowing that Y_i is at a distant r from the origin point, we can compute the area of the intersection $\nu(B'_0 \cap B'_{Y_i}) = A(r)$ where

$$A(r) = 2R^2 arccos\frac{r}{2R} - r\sqrt{R^2 - \frac{r^2}{4}}$$

and thus, since Y_i is uniformly distributed in B'_0, we have

$$\begin{aligned}\mathbb{E}^o\left[\nu(\Phi(B'_0 \cap B'_{Y_i}))\right] &= \mathbb{E}^o\left[A(r)\right] \\ &= \int_0^{2\pi}\int_0^R \frac{A(r)}{\pi R^2}\, r\, dr\, d\theta \\ &= R^2\left(\pi - \frac{3\sqrt{3}}{4}\right)\end{aligned}$$

Combined with equation 3, we obtain

$$
\begin{aligned}
&\mathbb{E}^o\left[\rho(0)\right] = \\
&\quad 1+\frac{1}{2}\sum_{k=1}^{+\infty}\sum_{i=1}^{k}\frac{1}{k}\frac{k-1}{\pi}\left(\pi-\frac{3\sqrt{3}}{4}\right)\mathbb{P}^o\left(\Phi(B'_0)=k\right) \\
&=1+\frac{1}{2}\sum_{k=1}^{+\infty}\frac{k-1}{\pi}\left(\pi-\frac{3\sqrt{3}}{4}\right)\mathbb{P}^o\left(\Phi(B'_0)=k\right) \\
&=1+\frac{1}{2\pi}\left(\pi-\frac{3\sqrt{3}}{4}\right) \\
&\qquad\times\left(\sum_{k=1}^{+\infty}k\mathbb{P}^o\left(\Phi(B'_0)=k\right)-\sum_{k=1}^{+\infty}\mathbb{P}^o\left(\Phi(B'_0)=k\right)\right) \\
&=1+\frac{1}{2\pi}\left(\pi-\frac{3\sqrt{3}}{4}\right)\left(\lambda\pi R^2-\left(1-exp\{\lambda\pi R^2\}\right)\right)
\end{aligned}
\tag{4}
$$

This last equality comes from Slyvniack's theorem which says in this case that $\Phi(B'_0)$, under Palm probability, follows a discrete Poisson distribution of parameter $\lambda\pi R^2$. ■

E. Analysis of the number of cluster heads

We first need some technical lemmas.

***Lemma** 2:* The average number of cluster heads that belongs to a given Borel subset C is given by:

$$
\begin{aligned}
&\mathbb{E}\left[\text{Number of heads in a Borel subset } C\right] \\
&\quad=\lambda\nu(C)\mathbb{P}^o\left(0 \text{ is head }\right)
\end{aligned}
\tag{5}
$$

***Lemma** 3:* The probability that the origin is a cluster head under Palm probability is given by:

$$
\mathbb{P}^o\left(0 \text{ is cluster head}\right) \;=\; \mathbb{P}^o\left(\rho(0)>\max_{k=1,..,\Phi(B_0)}\rho(Y_k)\right)
$$

thus where the sequence Y_k represents the points of Φ in B_0, the ball centered at the origin with a radius R. We fix $\max_{k=u,..,v}\rho(Y_k)=0$ if $v<u$.

We can now bound the quantity defined in Lemma 3.

***Theorem** 1:* An upper bound on the number of cluster head is given by:

$$
\begin{aligned}
&\mathbb{P}^o\left(\rho(0)>\max_{k=1,..,\Phi(B_0)}\rho(Y_k)\right) \\
&\quad\leq\left(1+\sum_{n=1}^{+\infty}\frac{1}{n}\frac{\left(\lambda\pi R^2\right)^n}{n!}\right)\exp\left\{-\lambda\pi R^2\right\}
\end{aligned}
\tag{6}
$$

Proof:

Let $B_0^{'}$ be the ball centered in 0 with a radius of R minus the singleton 0. In the case of the node at the origin point is the only one in B_0, it is obviously a cluster head. Indeed, it has the highest density value among nodes in B_0. We have.

$$
\begin{aligned}
&\mathbb{P}^o\left(\rho(0)>\max_{k=1,..,\Phi(B_0)}\rho(Y_k)\right)= \\
&\qquad\mathbb{P}^o\left(\rho(0)>\max_{k=1,..,\Phi(B_0)}\rho(Y_k)\Big|\Phi(B_0^{'})>0\right) \\
&\qquad\times\mathbb{P}^o\left(\Phi(B_0^{'})>0\right)+\mathbb{P}^o\left(\Phi(B_0^{'})=0\right)
\end{aligned}
$$

Thus, we compute:

$$
\begin{aligned}
p_0=&\mathbb{P}^o\left(\rho(0)>\max_{k=1,..,\Phi(B_0)}\rho(Y_k)\Big|\Phi(B_0^{'})>0\right) \\
&\times\mathbb{P}^o\left(\Phi(B_0^{'})>0\right)
\end{aligned}
$$

$\rho(Y_1)$ Palm distribution knowing that $\Phi(B_0^{'})>0$ where Y_1 is one of $B_0^{'}$'s nodes, is the same that the one of the density value of the node at the origin point, knowing that $\Phi(B_0^{'})>0$.

However, we have:

$$
\begin{aligned}
&p_0< \\
&\quad\mathbb{P}^o\left(\rho(Y_1)>\max(\rho(0),\max_{k=2,..,\Phi(B_0)}\rho(Y_k))\Big|\Phi(B_0^{'})>0\right)
\end{aligned}
$$

The proof of this inequality is omitted here and will be presented in futur paper.

Moreover, the event

$$
\{\rho(Y_1)>\max(\rho(0),\max_{k=2,..,\Phi(B_0)}\rho(Y_k))\}
$$

is included in the event

$$
\{\rho(Y_1)>\max_{k=2,..,\Phi(B_0)}\rho(Y_k))\}
$$

$$
\begin{aligned}
p_0\leq&\mathbb{P}^o\left(\rho(Y_1)>\max_{k=2,..,\Phi(B_0^{'})}\rho(Y_k)\Big|\Phi(B_0^{'})>0\right) \\
&\times\mathbb{P}^o\left(\Phi(B_0^{'})>0\right) \\
=&\sum_{n=1}^{+\infty}\mathbb{P}^o\left(\rho(Y_1)>\max_{k=2,..,\Phi(B_0^{'})}\rho(Y_k)\Big|\Phi(B_0^{'})=n\right) \\
&\times\mathbb{P}^o\left(\Phi(B'_0)=n\right) \\
=&\sum_{n=1}^{+\infty}\mathbb{P}^o\left(\rho(Y_1)>\max_{k=2,..,\Phi(B_0^{'})}\rho(Y_k)\Big|\Phi(B_0^{'})=n\right) \\
&\times\mathbb{P}\left(\Phi(B_0)=n\right) \\
\leq&\sum_{n=1}^{+\infty}\frac{1}{n}\frac{\left(\lambda\pi R^2\right)^n}{n!}\exp\left\{-\lambda\pi R^2\right\}
\end{aligned}
$$

The last equality is obtained thanks to the fact that the densities of the points standing in B_0 are equi-distributed since the locations of these points are uniformly and independantly distributed in B_0. More precisely,

$$\mathbb{P}^o\left(\rho(Y_i) > \max_{k=1,..,n;k\neq i} \rho(Y_k) \middle| \Phi(B_0^{'}) = n\right) \quad \leq \quad \frac{1}{n} \tag{7}$$

Moreover, the number of nodes under Palm distribution in a $\mathbb{R}^2$ Borel set which does not contain the origin point, follows a discrete Poisson law (Slivnyak's theorem [16] page 121).

We finally have:

$$\mathbb{P}^o\left(\rho(0) > \max_{k=1,..,\Phi(B_0)} \rho(Y_k)\right)$$
$$\leq \exp\left\{-\lambda\pi R^2\right\} + \sum_{n=1}^{+\infty} \frac{1}{n} \frac{\left(\lambda\pi R^2\right)^n}{n!} \exp\left\{-\lambda\pi R^2\right\}$$

■

F. Example

To illustrate this heuristic, let's take the following example (Fig. 1). Let's suppose that the node E is too mobile to be eligible.

In its 1-neighborhood topology, node A has two 1-neighbors ($\Gamma_1(\mathrm{A}) = \{D, I\}$) and two links ($\{(A, D), (A, I)\}$); Node B has 4 1-neighbors ($\Gamma_1(\mathrm{B}) = \{C, D, H, I\}$) and five links ($\{(B, C), (B, D), (B, H), (B, I), (H, I)\}$). Table I shows the final results.

In the illustrative example, node C joins its 1-neighbor which density is the highest: node B ($\mathcal{H}(C) = \mathcal{H}(B)$). Yet, the node with the highest density in node B's neighborhood is H. Thus, $\mathcal{H}(B) = \mathcal{H}(H)$ and so $\mathcal{H}(C) = \mathcal{H}(H)$. As node H has the highest density in its own neighborhood, it becomes its own cluster head: $\mathcal{H}(H) = H$. To sum up, C joins B which joins H and all three of them belong to the cluster which cluster head is H: $\mathcal{H}(C) = \mathcal{H}(C) = \mathcal{H}(H) = H$. Moreover, we have $\rho_1(J) = \rho_1(F)$. As it is the first construction, none of J and F was cluster head before. If we suppose that J has the smallest Id between both nodes $\mathcal{H}(F) = \mathcal{H}(J) = J$. At last, we obtain two clusters organized around two cluster heads: H and J. (See figure bellow on figure 1)

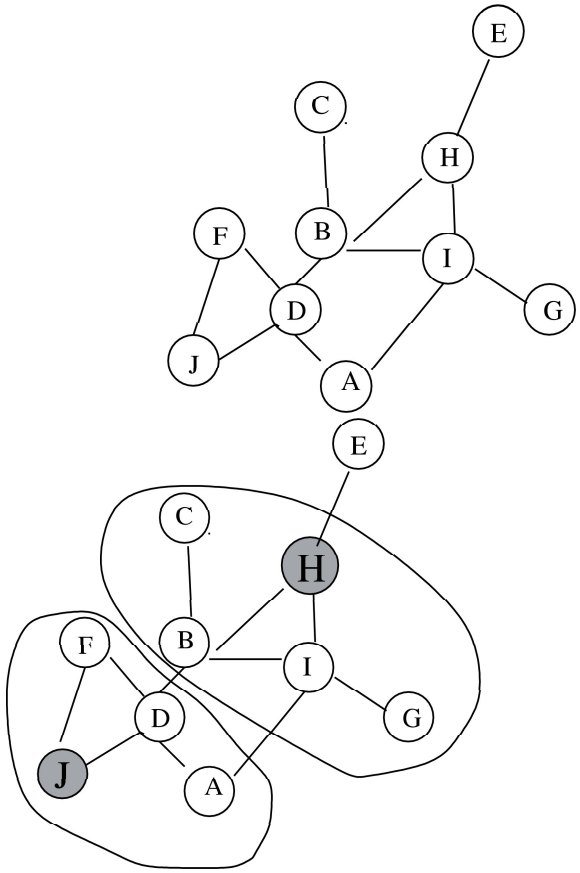

Fig. 1. Clustering example.

VI. Simulation and results

We performed simulations in order to evaluate the performance of the proposed heuristic and compare it with the Highest-Connectivity (Degree) [4] and the Max-Min d-clusters [1]. The geometric approach used in the analysis allows to model the spatial organization of networks. As in Section V-D, nodes are randomly deployed using a Poisson process in a 1×1 square ($1km^2$) with varying levels of intensities λ (and thus varying number of nodes) varying from $500km^{-2}$ to $1000km^{-2}$ which gives on average from 500 to 1000 nodes above our simulation square environment. Two nodes are said to have a wireless link between them if they are within communication range of each other. The communication range R is set to 0.1km in all tests. Some of the more noteworthy simulation statistics measured are: number of cluster heads, cluster diameter, nodes eccentricity in its cluster and cluster stability. These statistics provid a basis for evaluating the performance of the proposed heuristic. In each case, each statistic is the average over 1000 simulations. Note that as opposed to [1], for a given number of nodes, we fix a minimum radius such that the network is connected.

Results in Table II compare both theoretical analysis and simulated results of our heuristic for the average degree and nodes density. They match pretty well.

Nodes	A	B	C	D	E	F	G	H	I	J
Neighbors	2	4	1	4		2	1	2	4	2
Links	2	5	1	5		3	1	3	5	3
1-density	1	1.25	1	1.25		1.5	1	1.5	1.25	1.5

TABLE I

RESULTS OF OUR HEURISTIC ON THE ILLUSTRATIVE EXAMPLE.

	500 nodes		600 nodes		700 nodes	
	Theory	Simulation	Theory	Simulation	Theory	Simulation
mean degree	14.7	14.3	17.8	17.3	21.0	20.2
mean 1-density	4.7	5.0	5.6	5.9	6.5	6.8
	800 nodes		900 nodes		1000 nodes	
	Theory	Simulation	Theory	Simulation	Theory	Simulation
mean degree	24.1	23.1	27.3	25.9	30.0	29.0
mean 1-density	7.5	7.1	8.4	8.6	9.3	9.4

TABLE II

AVERAGE DEGREE AND DENSITY OF NODES.

	500 nodes	600 nodes	700 nodes	800 nodes	900 nodes	1000 nodes
Number of clusters	15	14.49	14.23	15.5	13.02	14
Number of nodes by cluster	31.2	41.4	49.2	51.5	69.1	72.7
$D(\mathcal{C})$	4.99	5.52	5.5	5.65	6.34	6.1
$\tilde{e}(u/\mathcal{C})$	2.1	2.3	2.3	2.4	2.4	2.6

TABLE III

CLUSTER CHARACTERISTICS FOR 1-DENSITY.

A. *Clusters characteristics*

Major characteristics of clusters and cluster heads are presented in table III. Note that our heuristic based on the 1-density is scalable: when the number of nodes significantly increases (from 500 to 1000) and the node eccentricity remains the same, the number of clusters is stable. Figure 2 compares experimental results and analytic upper bound of the number of clusters for an observation area 1*1 and $R = 0.1$.

In Figure 3, we compare the amount of clusters produced by our metric and by the Max-Min d-Clusters heuristic for $D = 3$ (the one the closest of ours as we have a mean cluster diameter around 6 hops) over a 1000 nodes topology for different values of R.

We then can see that the number of clusters formed by both metrics is similar when the radius is pretty high but that Max-Min d-Cluster computes more small clusters when the network is sparse. Thus, our metric has a better behavior towards sparse network (less connected) as it forms less clusters and then generates less control traffic. Moreover, at the opposite of Max-Min, our heuristic does not allow clusters with only one node (the cluster head) and the cluster head is pretty centered in its cluster.

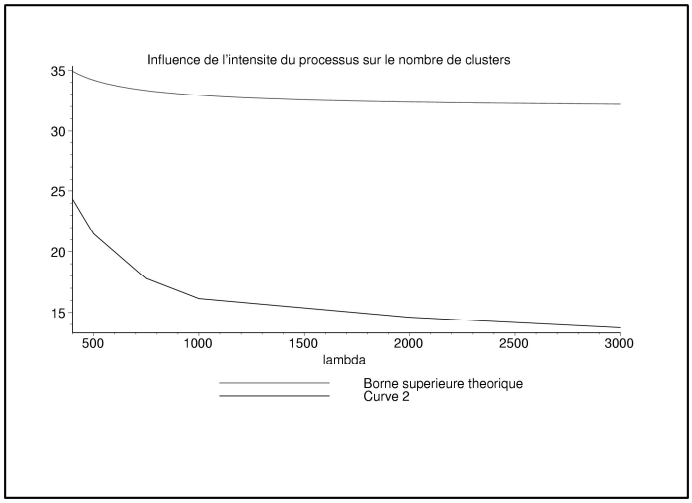

Fig. 2. Number of clusters in function of Poisson process intensity

Figures 4 and 5 plot one example of cluster organization results obtained during a simulation, both from the same nodes distribution. We can notice that clusters are homogeneous and correspond to what we expected:

	500 nodes		600 nodes		700 nodes	
	Density	Max Min	Density	Max Min	Density	Max Min
$\frac{\tilde{e}(\text{Cluster head}/\mathcal{C})}{D(\mathcal{C})}$	0.52	0.55	0.48	0.41	0.51	0.47
	800 nodes		900 nodes		1000 nodes	
	Density	Max Min	Density	Max Min	Density	Max Min
$\frac{\tilde{e}(\text{Clusterhead}/\mathcal{C})}{D(\mathcal{C})}$	0.47	0.39	0.48	.48	0.52	0.48

TABLE IV
CLUSTERS CHARACTERISTICS

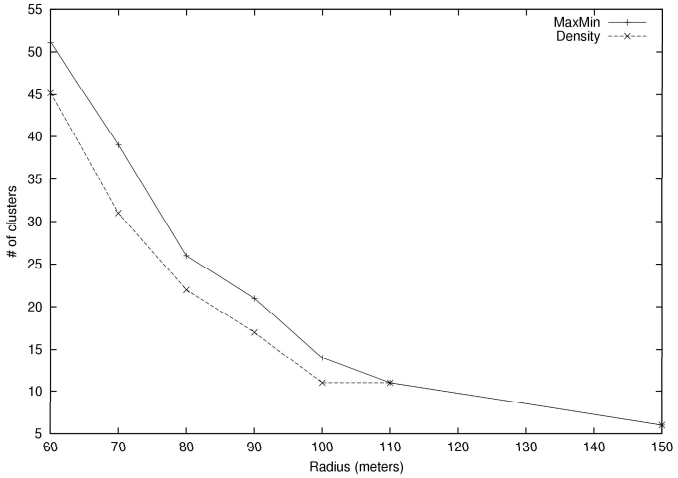

Fig. 3. Number of clusters produced for 1000 nodes in function of radius R by density metric $(- \times -)$ and Max Min 3-cluster metric $(- + -)$.

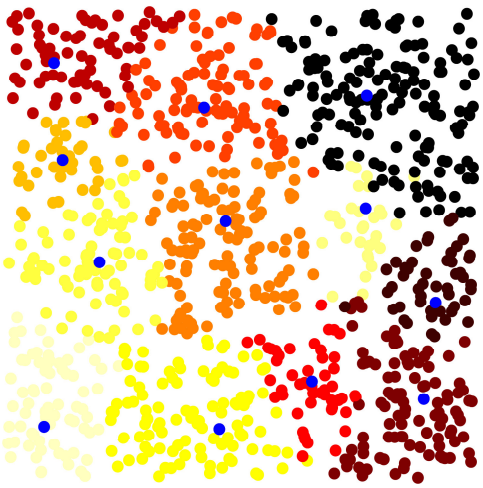

Fig. 4. Example of clusters organization for 1000 nodes with a radius $R = 0.1km$ with the 1-density heuristic.

cluster heads are well distributed over the environment in a homogeneous way. Clusters gather close nodes with high connectivity in order to favor intra-cluster traffic. We can observe that cluster heads are less well distributed with the Max-Min heuristic than with the density's one.

To illustrate the position of the cluster head in its cluster, we calculate ratio between the cluster head eccentricity in its own cluster and the diameter of the cluster. Results are presented in Table IV. They show that in the clusters organization resulting from our heuristic, cluster head are closer to the middle of the cluster than with the Max-Min's heuristic, which implies less control exchanges into a cluster.

B. Stability

In the aim to evaluate our metric over organization stability, we compare it with the degree heuristic (like in [4]). We expect the organization to change as less as possible, that means that the cluster heads remain cluster heads as long as possible. Indeed a cluster is defined

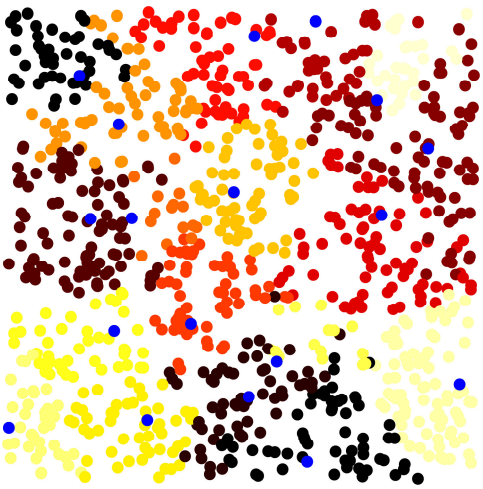

Fig. 5. Example of clusters organization for 1000 nodes with a radius $R = 0.1km$ with the Max Min 3-cluster.

by its cluster head, other nodes can migrate from one cluster to another one, this will not break the cluster. Then, the most noteworthy factor is number of changes among cluster heads.

Therefore, we perform simulations in which nodes can move in a random way at a random speed from 0 to $10m/s$ (for cars) and from 0 to $1.6m/s$ (for pedestrians). We observe each 2 seconds during 15 minutes. Results presented in Table V show that in average, our metric reconstructs clusters less often than the degree heuristic, it's thus better since more robust towards node mobility.

C. Non uniform distribution and arrival

The last test that we perform is when nodes are not uniformly distributed but rather concentrated around fews points, for example cities. Figures 6 and 7 illustrate such scenarii. As we can see our heuristic generates less clusters and clusterheads are much more centered inside their cluster. The Max Min heuristic generates sometime several useless neighbor cluster heads. For 1000 nodes, on average, our heuristics generates 8.7 clusters whereas the Max-Min heuristics generates 15.25 clusters.

Moreover, we also performed arrival tests, that is, as opposed to classical scenarii where nodes only move, we start the scenario with an initial configuration (800 nodes) and nodes arrives randomly in the network by groups of 50. As expected, Max-Min is less robust since the cluster head election is based on a purely static data, the ID of a node. Thus if one node vanishes or appears, it is enough to trigger a re-election and a modification in the cluster head which it is not the case in our heuristic since the density measure is able to "absorb" local modification.

Finally, Max-Min d-cluster algorithm is more costly in term of messages since it is composed of 3 phases that flood messages up to d hops in order to converge: one to compute the max, one to compute the min, and one to announce the winner. Our heuristic is purely local and we can implement it by doing piggy packing in `Hello` packets.

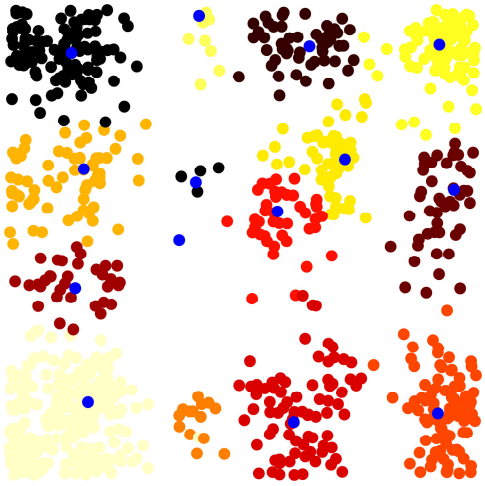

Fig. 6. Non uniform distribution of nodes: Example of clusters organization for 1000 nodes with a radius $R = 0.1km$ with the 1-density heuristic.

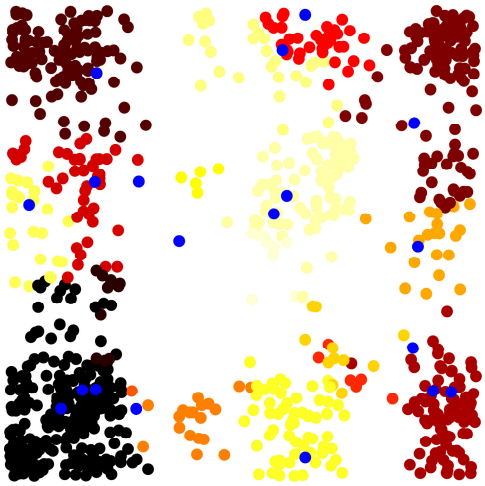

Fig. 7. Non uniform distribution of nodes: Example of clusters organization for 1000 nodes with a radius $R = 0.1km$ with the Max Min 3-cluster.

VII. Conclusion and Perspectives

We have proposed a distributed algorithm for organizing ad hoc (or sensor) nodes into a flexible hierarchy of clusters with a strong objective of not using fixed and non adaptive criteria. Thanks to stochastic geometry and Palm distribution theory, we have performed formal analysis and we were able to compute the average density of nodes but also we can bound the number of cluster heads in a given area if nodes are randomly distributed. We have shown by simulation and analytic analysis that our metric based on the density gathers the dynamics of node neighborhood and outperforms classical static criteria used in past solutions (e.g., max degree).

In future, we intend to test deeper our metric and its behavior over different environments. Our first results tend to show that formed clusters are closed to Voronoi tessellation and thus we can expect to derive promising properties of such organization. Indeed, it seems that a cluster head of a node is in fact the closest clusterhead. We are currently investigating the use of purely distributed hash function in order to solve the node localization problem once the clusterization is

	500 nodes		600 nodes		800 nodes		1000 nodes	
	Density	Degree	Density	Degree	Density	Degree	Density	Degree
speed from 0 to $1.6m/s$	68.7	65	67.2	63.5	64.5	62.4	62.2	56.8
speed from 0 to $10m/s$	30.1	27.5	27	25.3	26.2	23.1	24.8	20.35

TABLE V

% OF CLUSTERHEAD REELECTION FOR TWO DIFFERENT SPEEDS.

done. Once again, we should be able to apply stochastic geometry in order to derive formal bound on the number of hops (and not the euclidean distance) between nodes and their clusterhead.

REFERENCES

[1] A. Amis, R. Prakash, T. Vuong, and D. Huynh. Max-min d-cluster formation in wireless ad hoc networks. In *Proceedings of the IEEE INFOCOM*, march 2000.

[2] D. Baker and A. Ephremides. The architectural organization of a mobile radio network via a distributed algorithm. *IEEE Transactions on Communications*, 29(11):1694–1701, 1981.

[3] P. Basu, N. Khan, and T. Little. A mobility based metric for clustering in mobile ad hoc networks. In *Proceedings of Distributed Computing Systems Workshop 2001*, 2001.

[4] G. Chen and I. Stojmenovic. Clustering and routing in mobile wireless networks. Technical Report TR-99-05, SITE, June 1999.

[5] B. Das and V. Bharghavan. Routing in ad-hoc networks using minimum connected dominating sets. In *ICC*, 1997.

[6] Y. Fernandess and D. Malkhi. K-clustering in wireless ad hoc networks. In *Proceedings of the second ACM international workshop on Principles of mobile computing*, 2002.

[7] M. Gerla and J. Tzu-Chieh Tsai. Multicluster, mobile, multimedia radio network. *Baltzer Journals*, July 1995.

[8] A. Gibbons. *Algorithmic graph theory*. Cambridge University Press, 1985. ISBN 0-521-24659-8.

[9] M. Jiang and Y. Tay. Cluster based routing protocol(CBRP). DRAFT draft-ietf-manet-cbrp-spec-01.txt, IETF, July 1999.

[10] L. Kai and L. Jiandong. Mobile cluster protocol in wireless ad hoc networks. In *International Conference on Communication Technology (ICCT'2000) Proceedings*, august 2000.

[11] L. Kleinrock and J. Silvester. Spatial reuse in multihop packet radio networks. In *Proceedings of the IEEE*, volume 75, pages 156–167. IEEE, 1987.

[12] P. Krishna, N. H. Vaidya, M. Chatterjee, and D. K. Pradhan. A cluster based approach for routing in dynamic networks. In *ACM SIGCOMM*, pages 49–65. ACM, ACM, April 1997.

[13] H.-C. Lin and Y.-H. Chu. A clustering technique for large multihop mobile wireless networks. In *Proceedings of the IEEE Vehicular technology conference*, Tokyo, Japan, may 2000. IEEE.

[14] M. Pearlman, Z. Haas, and S. Mir. Using routing zones to support route maintenance in ad hoc networks. In *Wireless Communications and Networking Conference (WCNC 2000)*, pages 1280–1285. IEEE, September 2000.

[15] R. Ramanathan and M. Steenstrup. Hierarchically-organized, multihop mobile wireless networks for quality-of-service support. *Mobile networks and applications*, 3:101–119, June 1998.

[16] D. Stoyan, S. Kendall, and J. Mecke. *Stochastic geometry and its applications, second edition*. John Wiley & Sons, 1995.